1) (a) order 3

(b) linear

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2) $y = y_h + y_p = c_1 + c_2 e^x + 3x$

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3) $y' + 3x^2 y = 3x^2$

$A(x) = \int 3x^2\,dx = x^3$

$e^{x^3} y' + 3x^2 e^{x^3} y = 3x^2 e^{x^3}$

$(e^{x^3} y)' = 3x^2 e^{x^3}$

$e^{x^3} y = \underbrace{\int 3x^2 e^{x^3}\,dx}$

$\int e^u\,du = e^u + C = e^{x^3} + C$

$u = x^3$

$du = 3x^2\,dx$

$e^{x^3} y = e^{x^3} + C$

$$\boxed{y = 1 + Ce^{-x^3}}$$

4

$$\frac{dy}{dx} = 6y^2 x$$

$$y^{-2}\,dy = 6x\,dx$$

$$\int y^{-2}\,dy = \int 6x\,dx$$

$$\frac{y^{-1}}{-1} = 6\frac{x^2}{2} + C$$

$$-\frac{1}{y} = 3x^2 + C$$

$$\boxed{y = \frac{-1}{3x^2 + C}}$$

Need $y(0) = 1$:

$$1 = y(0) = \frac{-1}{3(0)^2 + C}$$

$$1 = \frac{-1}{C}$$

$$C = -1$$

$$\boxed{y = \frac{-1}{3x^2 + 1}}$$

5 (a) $\underbrace{(x^2+y^2)}_{M(x,y)} + \underbrace{(2xy)}_{N(x,y)}y' = 0$

$M(x,y) = x^2+y^2$

$N(x,y) = 2xy$

$\frac{\partial M}{\partial x} = 2x \qquad \frac{\partial M}{\partial y} = 2y$

$\frac{\partial N}{\partial y} = 2x \qquad \frac{\partial N}{\partial x} = 2y$

these are all continuous for all x,y. So take R to be the xy-plane

And, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

So, the equation is exact

(b) Need to solve

$\boxed{\begin{aligned}\frac{\partial f}{\partial x} &= x^2+y^2 \\ \frac{\partial f}{\partial y} &= 2xy\end{aligned}} \rightarrow \boxed{\begin{aligned} f(x,y) &= \tfrac{1}{3}x^3+y^2x+C(y) \quad (1) \\ f(x,y) &= xy^2+D(x) \quad (2)\end{aligned}}$

Set equal: $\frac{1}{3}x^3 + \cancel{y^2 x} + C(y) = \cancel{xy^2} + D(x)$

$$\frac{1}{3}x^3 + \underbrace{C(y)}_{0} = D(x)$$

Plug $C(x)=0$ into (1) to get $f(x,y) = \frac{1}{3}x^3+y^2x$

$\boxed{\text{Answer: } \frac{1}{3}x^3+y^2x = c \text{ where } c \text{ is a constant}}$

(6) $y'' - y' - 6y = 0$

$r^2 - r - 6 = 0$

$$r = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-6)}}{2(1)} = \frac{1 \pm \sqrt{25}}{2}$$

$$= \frac{1 \pm 5}{2} = \frac{6}{2}, \frac{-4}{2} = \boxed{3, -2}$$

$$\boxed{\underline{\text{Answer}}: \quad y = c_1 e^{3x} + c_2 e^{-2x}}$$